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...ulations Concerning the Passage of a Satellite Through the Earth's Shadow

by

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NASA

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Calculations Concerning the Passage of a Satellite Through the Earth's Shadow

by

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RESEARCH PROJECTS DIVISION

ABSTRACT

This report concerns the time a satellite spends in the shadow of the earth and is the result of work initiated by Gerhard Heller as part of the thermal control assignment for the EXPLORER satellites [4]. It deals with satellites in elliptical orbits inclined to the earth's equator. The derivation of the equations, the form in which they were given to the Computation Division, and the results of a particular case are presented.

TABLE OF CONTENTS

NOMENCLATURE

T_o	=	Universal Time during initial perigee passage
D	=	Number of days after vernal equinox
e	=	Eccentricity of orbit
LON	=	Longitude of initial perigee
LAT	=	Latitude of initial perigee
i	=	Inclination of orbital plane to equatorial plane
S	=	Sign of north velocity component at initial perigee (+ or -)
t	=	Time after initial perigee (days)
R_p	=	Radius of perigee
R	=	Radius vector to satellite
R_o	=	Radius of earth
h	=	Altitude of the satellite
a	=	Semi-major axis
P	=	Anomalistic period of satellite
z	=	Space-fixed azimuth at initial perigee
H_o	=	True sun time at initial perigee
β	=	Angle between radius vector to satellite and radius to point at the surface of the earth where a line from the satellite to the earth would be tangent.
Ω_{oo}	=	Angle between perigee and ascending node in plane of equator

ω = Argument of perigee

E_O = Angle between equator and ecliptic in plane of orbit

γ = Declination of the sun with respect to the equatorial plane

δ = Declination of the sun with respect to the orbital plane

α = Right ascension of sun

L_Θ = Right ascension of sun in plane of ecliptic

Ω = Right ascension of ascending node

$\dot{\Omega}$ = Regression rate of ascending node along equator

Ω_Θ = Right ascension of ascending node in plane of ecliptic

ε = Inclination of plane of ecliptic to plane of equator

j = Inclination of orbital plane to ecliptic plane

ϕ = Angle between perigee and the twilight line in plane of orbit

σ = Angle between twilight line and perigee in plane of orbit

B = Angle between the intersection of the great circle perpendicular to the sun with the orbit and with the equator

$z^{\bullet}$ = Angle between the plane of the orbit and the great circle perpendicular to the sun

X = Angle between twilight line and ingress (X_2) or egress (X_1) in plane of orbit

m = Point in orbit nearest shadow axis

q = Projection of point, m , onto shadow axis

p = Projection of ingress point onto axis of shadow cylinder

θ	=	True anomaly of any point in orbit
ν	=	True anomaly of egress (ν_1) or ingress (ν_2)
E	=	Eccentric anomaly of egress (E_1) or ingress (E_2)
M	=	Mean anomaly of egress (M_1) or ingress (M_2)
T_x	=	Per cent time satellite spends in sunlight during each revolution
PS	=	Angle between perigee and sun in plane of orbit
AS	=	Angle between ascending node and sun in plane of orbit
T_E	=	Time from ascending node to egress
T_I	=	Time from, ascending node to ingress
T_p	=	Time from ascending node to perigee
$K_{1,2}$	=	Constants used in first terms of a series relating to the oblateness of the earth [2]
K_3	=	Constant from Kepler's third law
K_4	=	Mean angular motion of the earth around the sun
K_5	=	Equation of time at the vernal equinox [3]
$K_{6,7}$	=	Constants used in first terms of a series converting mean anomaly into true anomaly
K_8	=	True longitude of sun at perihelion can be expressed as a function of the year as shown in the calculations

1. INTRODUCTION

As a satellite travels around the earth in an elliptical orbit, it usually passes through the earth's shadow some time during each revolution, depending on the characteristics of the orbit (Fig. 1). The percentage of time that a satellite spends in the earth's shadow during each revolution can vary from 40 to 0 per cent. This shadow passage is very important since it affects either directly or indirectly the temperature of the satellite (Figure 2), the power supply of the satellite (if it is a type that utilizes solar energy), the tracking by visual means, and the experiments themselves. The per cent time spent in the shadow during each revolution is a function of the orbital elements and is continually changing due to the motion of the earth about the sun and the various perturbations on the orbit, primarily those caused by the oblateness of the earth.

Section 2 contains the derivation of equations used to calculate the per cent time spent in shadow or sunlight during each revolution, and other useful information.

In Section 3, the equations are presented step by step, just as they were given to the Computation Division for programming.

Section 4 gives some of the results of the computer program.

2. DERIVATIONS

Here, equations are derived for calculating the amount of time spent in shadow, the points in the orbit where the satellite enters and leaves the shadow and other information using the following data:

(1) the eccentricity, inclination, and radius of perigee of the orbit;

(2) the latitude, longitude, and time of a perigee passage; and

(3) whether the satellite is traveling north or south at perigee.

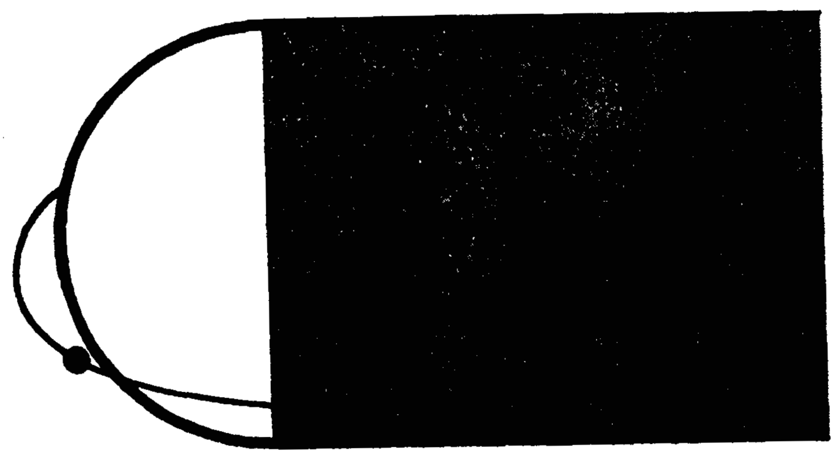
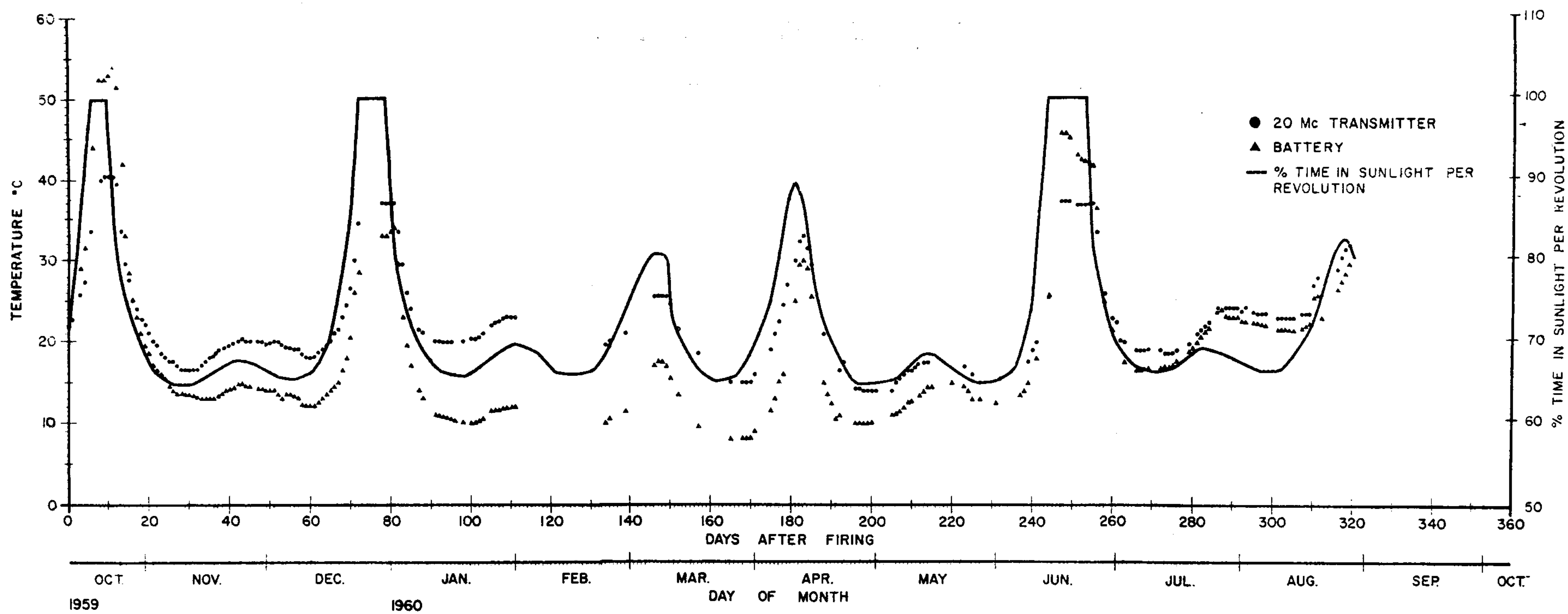


Fig. I Satellite orbit showing passage through shadow.

FIG. 2 Internal Temperatures and Per Cent Time Spent in Sunlight Per Revolution for Explorer VII



The motion of the earth about the sun and the perturbations on the orbit caused by the oblateness of the earth are also included so that computations can be made for as many days after the initial perigee as the orbital elements, e, R_p, and i, can be considered unchanged.

From Figures 3 through 9, we can obtain the relationship:

$$\cos S_2 = \cos \delta_2 \cos X_i$$

since

$$S_2 = 90 - \beta_2$$

and

$$X_i = 90 - X_2$$

then

$$\sin \beta_2 = \cos \delta_2 \sin X_2$$

or

$$\sin X_2 = \frac{\sin \beta_2}{\cos \delta_2} \quad \text{for ingress} \quad [1].$$

The same is also true for egress so that

$$\sin X_1 = \frac{\sin \beta_1}{\cos \delta_1} \quad .$$

We shall now derive the equations necessary to express the equation, $\sin X = \sin \beta / \cos \delta$, as a function of T_o, D_o, e, LON, LAT, i, R_p, and S.

Now $\cos \beta = R_o/R$ so that $\sin \beta = \sqrt{1 - \frac{R_o^2}{R^2}}$.

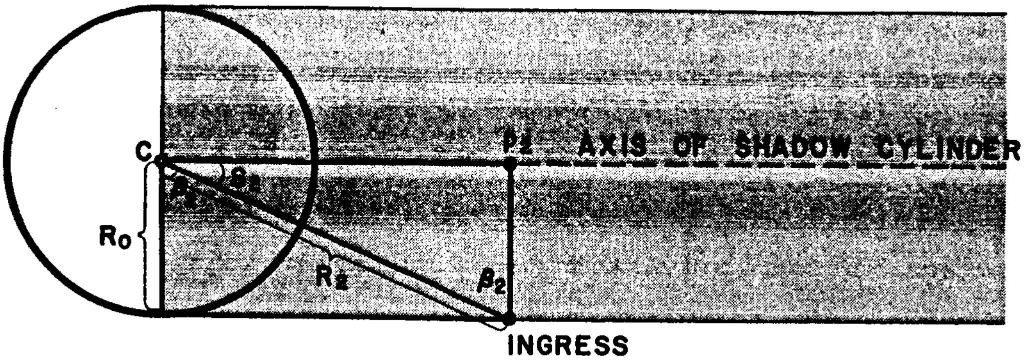


Fig. 3 Plane of shadow axis and radius vector to ingress point.

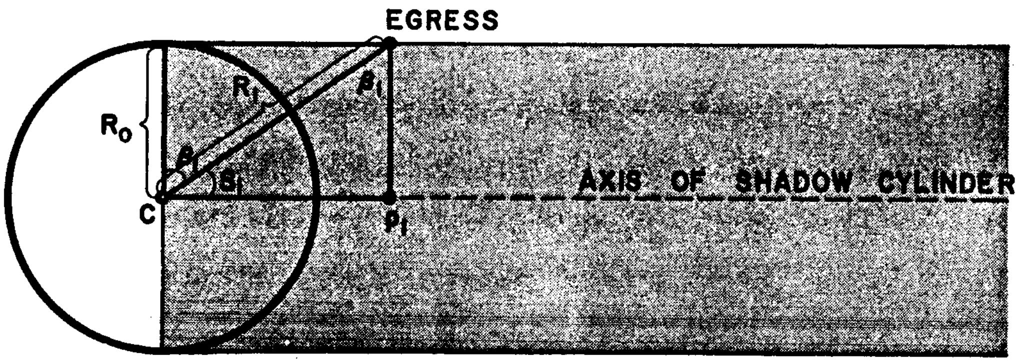


Fig. 4 Plane of shadow axis and radius vector to egress point.

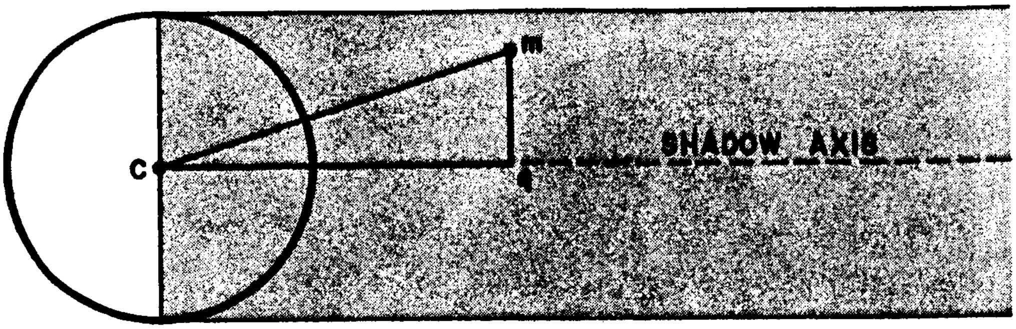


Fig. 5 Plane of shadow axis and radius vector to satellite at point m, in plane of orbit nearest shadow axis.

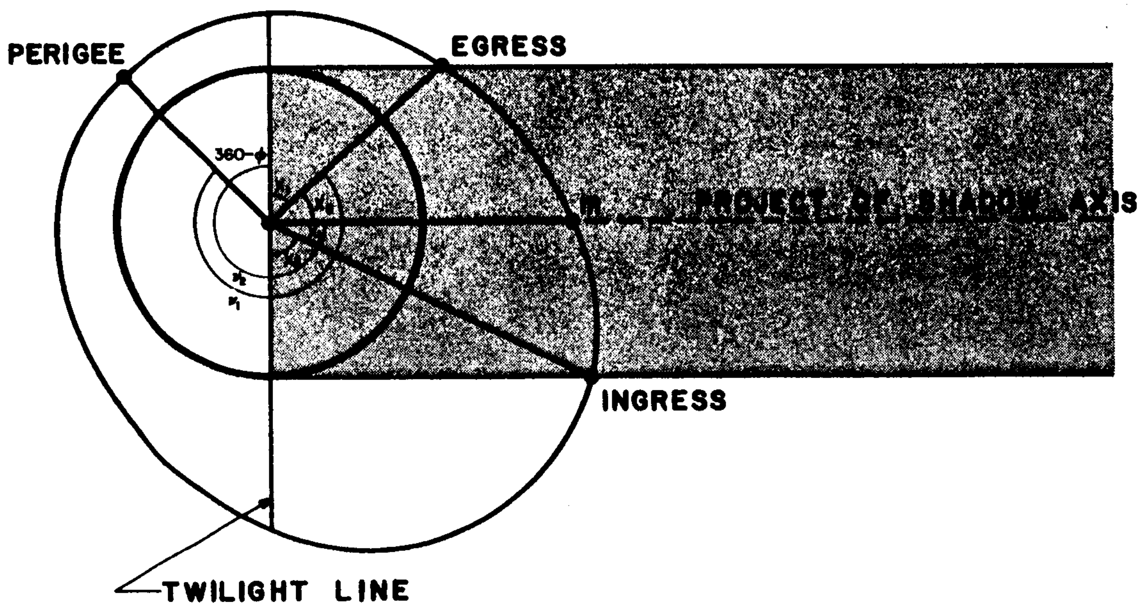


Fig. 6 Plane of orbit.

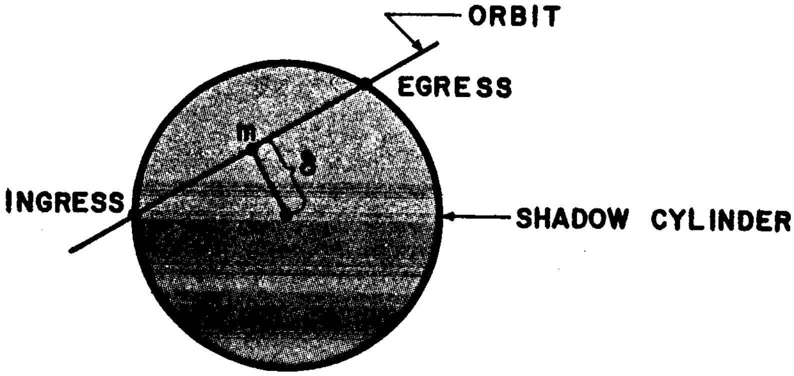


Fig. 7

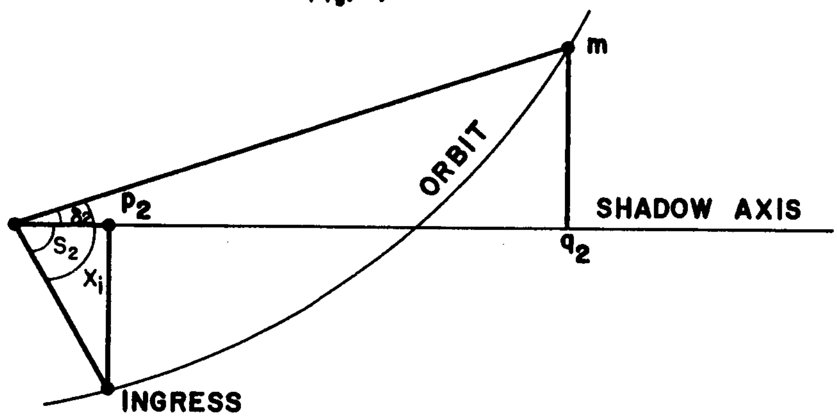


Fig. 8

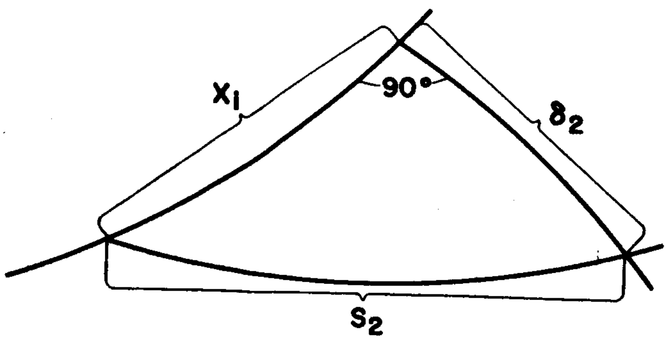


Fig. 9

From Kepler's equation,

$$R = \frac{a(1-e^2)}{1+e\cos\theta}$$

where $a = \dfrac{R_p}{1-e}$

so that

$$\sin\beta = \sqrt{1 - \left(\frac{R_o}{R_p}\right)^2 \left(\frac{1+e\cos\theta}{1+e}\right)^2}$$

and, therefore,

$$\sin X = \frac{\sqrt{1 - \left(\frac{R_o}{R_p}\right)^2 \left(\frac{1+e\cos\nu}{1+e}\right)^2}}{\cos\delta} .$$

From Fig. 10,

$$\sin\delta = \sin j \sin(\Omega_\Theta - L_\Theta) .$$

From Fig. 11,

$$\cos j = \cos i \cos\varepsilon + \sin i \sin\varepsilon \cos\Omega$$

and $\cos\Omega_\Theta = \cos E_o \cos\Omega - \sin E_o \sin\Omega \cos i$

where $\sin E_o = \sin\varepsilon \sin\Omega / \sin j$

and $\Omega = \Omega_o + \dot{\Omega} t$

where [2]

$$\dot{\Omega} = -\frac{2\pi}{P}\cos i \left\{3\frac{K_1}{[R_p(1+e)]^2} + 10\frac{K_2}{[R_p(1+e)]^4}\left(1+\frac{3}{2}e^2\right)\left(1-\frac{7}{4}\sin^2 i\right)\ldots\right\},$$

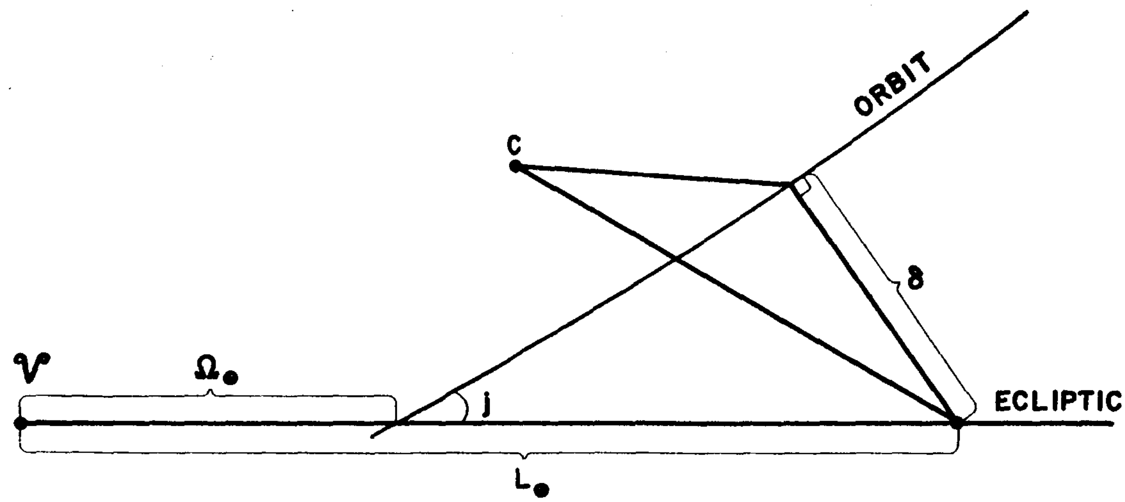


Fig. 10

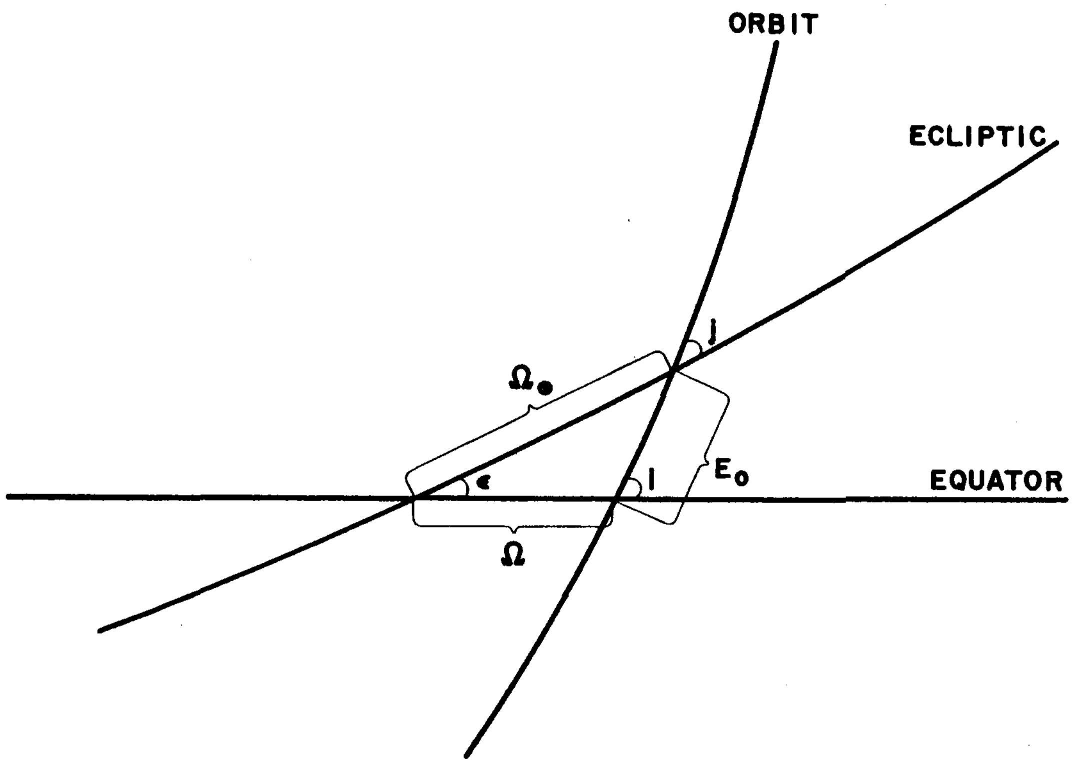


Fig. 11

and $P \approx K_3 \left[\frac{R_p}{1 - e}\right]^{\frac{3}{2}}$

From Fig. 12,

$$\Omega_o = \Omega_{oo} + \sigma_o - 90° + \alpha_o ,$$

where

$$\Omega_{oo} = \tan^{-1}[- \sin (LAT) \tan z]$$

$$\sigma_o = 15 (H_o - 6)$$

and

$$z = \sin^{-1} \left[\frac{\cos i}{\cos LAT}\right] .$$

Also, $H_o \approx T_o - \frac{LON}{15} + \frac{K_4 D_o - \alpha_o}{15} + K_5$

where the term $K_4 D_o - \alpha_o/15 + K_5$ is used to convert mean solar time into true solar time.

From Fig. 13,

$$\alpha = \tan^{-1} [\tan L_\Theta \cos \varepsilon]$$

where

$$L_\Theta \approx K_4 (D+77) + K_6 \sin [K_4 (D + 77)]$$

$$+ K_7 \sin 2[K_4 (D+77)] - (360 - K_8)$$

From Fig. 6, it can be seen that

$$\nu_2 = \phi - 180 + X_2$$

$$\nu_1 = \phi - X_1 ,$$

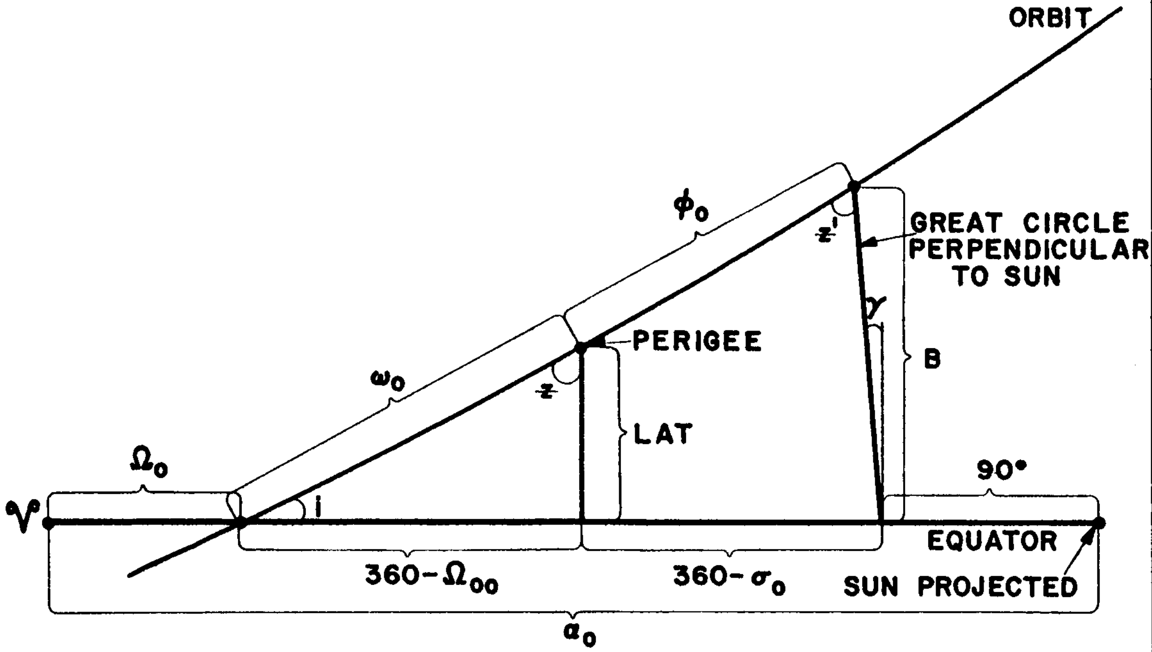

ORBIT
ϕ_0
GREAT CIRCLE PERPENDICULAR TO SUN
z'
γ
PERIGEE
ω_0
z
B
LAT
Ω_0
90°
Υ
i
EQUATOR
360-Ω_{00}
360-σ_0
SUN PROJECTED
α_0

Fig. 12

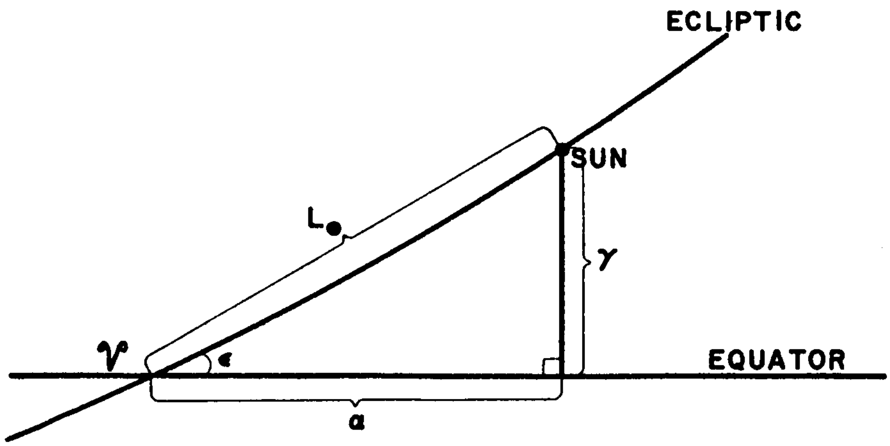

ECLIPTIC
SUN
$L_\odot$
γ
Υ
ϵ
EQUATOR
α

Fig. 13

and from Fig. 12,

$$\phi = \cos^{-1}[\cos B \cos(\alpha - \Omega - 90)$$

$$+ \sin B \sin(\alpha - \Omega - 90) \sin \gamma] - \omega,$$

where

$$B = \sin^{-1}\left[\sin i \ \frac{\sin(\alpha - \Omega - 90)}{\sin z'}\right]$$

$$z' = \cos^{-1}\{-\cos i \sin\gamma + [\sin i \cos\gamma$$

$$\cos(\alpha - \Omega - 90)]\} .$$

From Fig. 13,

$$\gamma = \sin^{-1}[\sin\varepsilon \sin\alpha] .$$

Now $\omega = \omega_o + \dot{\omega}\, t$

where [2]

$$\dot{\omega} = \frac{2\pi}{P}\left[3 \ \frac{K_1}{[R_p(1+e)]^2}\left(1 - \frac{3}{2}\sin^2 i\right)\right.$$

$$\left. + 10 \ \frac{K_2}{[R_p(1+e)]^4}\left(1 + \frac{3}{4}e^2\right)\left(1 - 5\sin^2 i + \frac{35}{8}\sin^4 i\right)\right] - \dot{\Omega}\cos i ,$$

and from Fig. 12,

$$\omega_o = 360 - \tan^{-1}\left[\frac{\tan LAT}{\cos(180 - z)}\right]$$

Using these equations, we are able to express the original equation, $\sin X = \sin\beta / \cos\delta$, as a function of the parameters, T_o, D_o, e, LON, LAT, i, R_p, and S only. This equation is transcendental; it may be solved by iteration or by graphical means.

After solving for the angles X_1 and X_2, we can obtain the true anomaly of egress and ingress by the relationships:

$$\nu_1 = \phi - X_1$$

$$\nu_2 = \phi - 180 + X_2 ,$$

from which we can obtain the eccentric anomaly by

$$E = 2 \tan^{-1}\left[\sqrt{\frac{1 - e}{1 + e}} \tan \frac{\nu}{2}\right] ,$$

and then the mean anomaly by

$$M = E - e \sin E ,$$

which can give us the per cent time the satellite spends in sunlight per revolution, T_x :

$$T_x = \left[\frac{M_2 - M_1}{360}\right] 100 .$$

The time it takes for the satellite to go from the ascending node to perigee may be calculated by

$$T_p = \frac{p}{2\pi}\left[E_p - e \sin E_p\right] ,$$

where

$$E_p = 2 \tan^{-1}\left[\sqrt{\frac{1 - e}{1 + e}} \quad \tan \frac{\omega}{2}\right] ,$$

and the time from ascending node to ingress and egress

$$T_I = T_p + \frac{P}{2\pi} M_2$$

$$T_E = T_p + \frac{P}{2\pi} M_1 .$$

We can calculate other angles which are useful, such as the angle between the ascending node and the projection of the sun in the plane of the orbit,

$$AS = \tan^{-1}\left[\tan (L_\odot - \Omega_\odot) \cos j\right. \\ \left. + \sin^{-1}\left[\frac{\sin \varepsilon \ \sin \Omega}{\sin j}\right]\right] ,$$

the angle between the perigee and the projection of the sun in the plane of the orbit,

$$PS = AS - \omega ,$$

and the angle between the sun and the plane of the orbit,

$$\delta = \sin^{-1} [\sin j \ \sin (\Omega_\odot - L_\odot)] .$$

3. COMPUTATION

Presented here are the equations as they were given to the Computation Division and programmed on the IBM 7090 by Mrs. Billie Robertson.

ORDER OF CALCULATION

1. $P = P(R_p, e)$
2. $z = z(i, \text{LAT})$
3. $\omega_o = \omega_o(\text{LAT}, z)$
4. $L_\Theta = L_\Theta(D, D_o, t)$
5. $\alpha = \alpha(L_\Theta, \varepsilon)$
6. $H_o = H_o(D_o, \alpha_o, T_o, \text{LON})$
7. $\sigma_o = \sigma_o(H_o)$
8. $\Omega_{oo} = \Omega_{oo}(\text{LAT}, z)$
9. $\Omega_o = \Omega_o(\Omega_{oo}, \alpha_o, \sigma_o)$
10. $\gamma = \gamma(\varepsilon, \alpha)$
11. $\dot{\Omega} = \dot{\Omega}(i, e, R_p, P)$
12. $\dot{\omega} = \dot{\omega}(i, e, R_p, P, \dot{\Omega})$
13. $\Omega = \Omega(\Omega_o, \dot{\Omega}, t)$
14. $\omega = \omega(\omega_o, \dot{\omega}, t)$
15. $z' = z'(i, \gamma, \alpha, \Omega)$
16. $B = B(i, \alpha, \Omega, z')$

17. $\phi = \phi\,(B, \alpha, \Omega, \gamma, \omega)$

18. $AS = AS\,(L_\Theta, \Omega_\Theta, j, \varepsilon, \Omega)$ $j = j(i, \varepsilon, \Omega)$ $\Omega_\Theta = \Omega_\Theta(\Omega, i, j)$

19. $X_{1,2} = X_{1,2}\,(R_O, R_p, e, j, \Omega_\Theta, L_\Theta, \phi, X_{1,2})$

20. $\nu_1 = \nu_1(\phi, X_1)$

21. $\nu_2 = \nu_2\,(\phi, X_2)$

22. $E_1 = E_1\,(e, \nu_1)$

23. $E_2 = E_2\,(e, \nu_2)$

24. $M_1 = M_1\,(E_1, e)$

25. $M_2 = M_2\,(E_2, e)$

26. $T_x = T_x\,(M_1, M_2)$

27. $T_p = T_p\,(P, E_p, e)$ $E_p = E_p\,(e, \omega)$

28. $T_I = T_I\,(T_p, P, M_2)$

29. $T_E = T_E\,(T_p, P, M_1)$

30. $PS = PS\,(AS, \omega)$

31. $\delta = \delta\,(j, \Omega_\Theta, L_\Theta)$

32. $LAT\ p = LAT\ p\,(i, \omega)$

33. PRINT

GIVEN:

T_o	Universal Time during initial perigee passage (hours)
D_o	Days after varnal equinox
e	Eccentricity
LON	Longitude of initial perigee (° west)
LAT	Latitude of initial perigee
i	Inclination of orbit
R_p	Radius of perigee (km)
S	Sign of north velocity component at perigee (+ or -)

\+ other constants which will be the same in all cases.

1. Calculate period, P

$$P \approx (2.764\ 10^{-6}) \left[\frac{R_p}{1 - e}\right]^{\frac{3}{2}} \text{hours}$$

2. Calculate space-fixed azimuth of perigee, z

$$z = \sin^{-1}\left[\frac{\cos i}{\cos \text{LAT}}\right] \text{radians}$$

where z is in the first quadrant
if S is +
and in the second quadrant
if S is - .

3. Calculate argument of perigee, ω_o at $t = 0$

$$\omega_o = \pi - \tan^{-1}\left[\frac{\tan LAT}{\cos(180 - z)}\right] \text{ radians}$$

where the " $\tan^{-1}$ " must be in the first or second quadrant if LAT is positive, and in the third or fourth quadrant if LAT is negative.

4. Calculate right ascension of sun in plane of ecliptic, L_Θ

$$L_\Theta = .9856(D+77) + 1.9481 \sin[.9856(D+77)]$$
$$+ .0207 \sin 2[.9856(D+77)] - (360 - K_8)$$

where $D = D_o + t + \dfrac{T_O}{24}$

and t is the number of days after D_o,

L_Θ will be in degrees.

The term (360 - K) may be expressed as a function of the year, as follows:

$$(360 - K) \approx 77.6^\circ + \left(\frac{[90^\circ(\text{year} - 1)]}{360}\right)$$

where the angle $[90^\circ (\text{year} - 1)]$ must be placed in the first revolution.

5. Calculate right ascension of sun in plane of equator, α_o , α; α_o is α for $t = 0$.

$$\alpha = \tan^{-1} [\tan L_\Theta \cos \varepsilon] \text{ radians}$$

$$\varepsilon = 23.45$$

where α is in same quadrant as L_Θ .

6. Calculate true sun time at perigee, H_o

$$H_o = \left| T_o - \frac{LON}{15} \right| + \frac{(.9856\, D_o - \alpha_o)}{15}$$

$$- .125 \text{ hours}$$

where the value of K_s for 1958 is -0.125.

7. Calculate the angle between the twilight line and perigee in plane of equator, σ_o

$$\sigma_o = 15\,(H_o - 6)$$

8. Calculate the angle between perigee and ascending node in plane of equator, Ω_{oo}

$$\Omega_{oo} = \tan^{-1}[-\sin(LAT)\tan z] \text{ radians}$$

where Ω_{oo} is in same quadrant as $2\pi - \omega_o$.

9. Calculate right ascension of ascending node in plane of equator, Ω_o at time $t = 0$.

$$\Omega_o = \Omega_{oo} + \alpha_o + \sigma_o - \frac{\pi}{2} \quad \text{radians}$$

10. Calculate the angle between the sun and the equator, γ

$$\gamma = \sin^{-1}[\sin\varepsilon \sin\alpha] \text{ radians}$$

where γ is in the first or fourth quadrants.

11. Calculate the regression of the ascending node, $\dot{\Omega}$ (rad/day)

$$\dot{\Omega} = -\frac{2\pi}{P}(24)\cos i\left[3\frac{K_2}{\bar{p}^2} + 10\frac{K_4}{\bar{p}^4}\left(1+\frac{3}{2}e^2\right)\left(1-\frac{7}{4}\sin^2 i\right)\right]$$

where $K_2 = 2.22256 \times 10^4$

$K_4 = 1.580570 \times 10^9$

$\bar{p} = R_p(1+e)$

12. Calculate the advance of perigee, $\dot{\omega}$

$$\dot{\omega} = \frac{2\pi}{P}(24)\left[3\frac{K_2}{\bar{p}^2}\left(1-\frac{3}{2}\sin^2 i\right) + 10\frac{K_4}{\bar{p}^4}\left(1+\frac{3}{4}e^2\right)\left(1-5\sin^2 i+\frac{35}{8}\sin^4 i\right)\right] - (\cos i)(\dot{\Omega})\ \frac{\text{radians}}{\text{day}}$$

13. Calculate right ascension of ascending node in plane of equator,

$$\Omega = \Omega_o + \dot{\Omega}t \text{ degrees}$$

14. Calculate the argument of perigee,

$$\omega = \omega_o + \dot{\omega}t \text{ degrees}$$

15. Calculate the angle between the plane of the orbit and the twilight line, z'

$$z' = \cos^{-1}\left[-\cos i \sin\gamma + (\sin i \cos\gamma \cos[\alpha - \Omega - 90])\right] \text{radians}$$

where z' is in the first or second quadrants

16. Calculate angle between orbit and equator in plane of twilight line, B

$$B = \sin^{-1}\left[\sin i \quad \frac{\sin(\alpha - \Omega - 90)}{\sin z'}\right] \text{radians}$$

where B is in the first or fourth quadrants.

17. Calculate angle between perigee and twilight line in plane of orbit, ϕ

$$\phi = \cos^{-1}[\cos B \cos(\alpha - \Omega - 90)$$
$$+ \sin B \sin(\alpha - \Omega - 90)\sin \gamma] - \omega \text{ radians}$$

Reduce to first revolution.

Set $(\alpha - \Omega - 90)$ so that it is between $0°$ and $360°$, then:

if $0 \leq (\alpha - \Omega - 90) \leq 180$

the "$\cos^{-1}$" is in the 1st or 2nd quadrant

if $180 \leq (\alpha - \Omega - 90) \leq 360$

the "$\cos^{-1}$" is in the 3rd or 4th quadrant

18. Calculate angle between ascending node and the projection of the sun in plane of orbit, AS

$$AS = \tan^{-1}\left[\tan(L_\odot - \Omega_\odot)\cos j\right]$$
$$+ \sin^{-1}\left[\frac{\sin \varepsilon \sin \Omega}{\sin j}\right]$$

where the $\tan^{-1}$ is in the same quadrant as $(L_\odot - \Omega_\odot)$

and $\sin^{-1}$ is in the first or fourth quadrants, and where

$$j = \cos^{-1}[\cos i \ \cos \varepsilon + \sin i \ \sin\varepsilon \cos \Omega]$$

where
$\cos^{-1}$ must be in the first or second quadrant.

Also,

$$\Omega_\theta = \cos^{-1}[\cos E_o \ \cos \Omega - \sin E_o \sin \Omega \cos i]$$

and

$$\Omega_\theta = \sin^{-1}\left[\frac{\sin\Omega \sin i}{\sin j}\right]$$

where

$$E_o = \sin^{-1}\left[\frac{\sin \varepsilon \ \sin \Omega}{\sin j}\right]$$

if $i > \varepsilon$, then E_o is in the first or fourth quadrant.

19. Calculate X_1 and X_2

$$\sin X_{1,2} = \sqrt{\frac{1 - \left[\frac{R_o (1 + e \cos\nu)}{R_p (1 + e)}\right]^2}{1 - \sin^2 j \ \sin^2 (\Omega_\theta - L_\theta)}}$$

for X_1 $\quad \nu = \phi - X_1$

X_2 $\quad \nu = \phi + X_2 + 180$

the $\sin^{-1}$ is in the first quadrant.

This equation is solved by iteration using the following information: if the right and left side of the equation are each plotted as a function of X, as X goes from 0° to 90°, it will be noted that the curve for the right side starts above the curve for the left side, and if there is a solution, they will intersect before they get to 90°. If there is no solution, then X_1 and X_2 should be set equal to 90°.

20. Calculate angle between perigee and point of egress (true anomaly of point of egress), ν_1

$$\nu_1 = \emptyset - X_1$$

reduce to first revolution.

21. Calculate true anomaly of ingress, ν_2

$$\nu_2 = \emptyset + 180 + X_2$$

reduce to first revolution.

22. Calculate eccentric anomaly of egress

$$E_1 = 2 \tan^{-1}\left[\sqrt{\frac{1-e}{1+e}} \tan \frac{\nu_1}{2}\right] \text{ radians}$$

where "$\tan^{-1}$" is in the same quadrant as $\nu_1/2$.

23. Calculate eccentric anomaly of ingress

$$E_2 = 2 \tan^{-1}\left[\sqrt{\frac{1-e}{1+e}} \tan \frac{\nu_2}{2}\right] \text{ radians}$$

where "$\tan^{-1}$" is in the same quadrant as $\nu_2/2$.

24. Calculate the mean anomaly of egress, M_1

$$M_1 = E_1 - e \sin E_1$$

25. Calculate mean anomaly of ingress, M_2

$$M_2 = E_2 - e \sin E_2 \text{ radians}$$

26. Calculate T_x

for $M_1 < M_2$ $\qquad T_x = \left(\frac{M_2 - M_1}{2\pi} \right) 100$

for $M_1 > M_2$ $\quad T_x = \frac{(M_2 - M_1 + 2\pi)}{2\pi} \; 100$

if $|M_1 - M_2| < .001$ radian

set $T_x = 100$

27. Calculate time from ascending node to perigee in minutes, T_p

$$T_p = \frac{P}{2\pi}(60) [E_p - e \sin E_p]$$

where $E_p = 2 \tan^{-1} \left[\sqrt{\frac{1 - e}{1 + e}} \tan \frac{\omega}{2} \right]$

and $\tan^{-1}$ is in the same quadrant as $\omega/2$.

28. Calculate time from ascending node to ingress, T_I

$$T_I = T_p + \frac{P}{2\pi}(60) M_2$$

29. Calculate time from ascending node to egress, T_E

$$T_E = T_p + \frac{P}{2\pi}(60) M_1$$

30. Calculate angle between perigee and the projection of the sun in plane of orbit, PS

$$PS = AS - \omega$$

31. Calculate angle between sun and plane of orbit, δ

$$\delta = \sin^{-1} [\sin j \sin (\Omega_{\theta} - L_{\theta})] \text{ radians}$$

where δ is in the first or fourth quadrant.

32. Calculate latitude of perigee, LAT P

$$\text{LAT P} = \sin^{-1}[\sin i \sin \omega] \quad \text{radians}$$

where LAT P is in the first or fourth quadrant.

33. Print initial conditions.

T_o must be varied from T_i to T_f in steps of ΔT.

For each "T_o" t varies from 0 to t_f in steps of Δt.

For each step of t all calculations, 1 through 32, must be done except 1, 2, 3, 6, 7, 8, 9, 11, and 12.

For each t print:

Day - (t)

T_x -

T_p -

T_I -

T_E -

AS -

δ -

Ω -

ω -
LAT P -

4. RESULTS

Some results from the computer program, particularly Explorers VII and VIII, are given using the initial orbital data and assuming that the orbital elements (R_p, e, and i) do not change appreciably during the period of time for which the calculations were made.

A page of the computer printout together with the nomenclature follows and Figures 14 and 15 show the plotted results. The time is referenced to the ascending node because, generally, the times of nodal crossings are readily available.

One immediate use for this type of graph is in the tracking of the satellite. A tracking station in a fixed geographic position can "see" only a certain part of the orbit. The two lines on each of the graphs show the part of the orbit which could usually be seen by the Huntsville tracking station some time during each day. By using this type of graph, the tracking station could be told what days to track in order to obtain such points of interest as ingress, egress, or perhaps both, as well as any other part of the orbit [5].

When analyzing data from a satellite, it is only necessary to know the time the data was taken and the time of the last ascending node to obtain the position of the satellite with respect to the earth's shadow and perigee.

Preliminary studies with this type of data allow the selection of the satellite launch time to permit a certain control over the shadow passages of the satellite. This is extremely useful when one considers the previously discussed points.

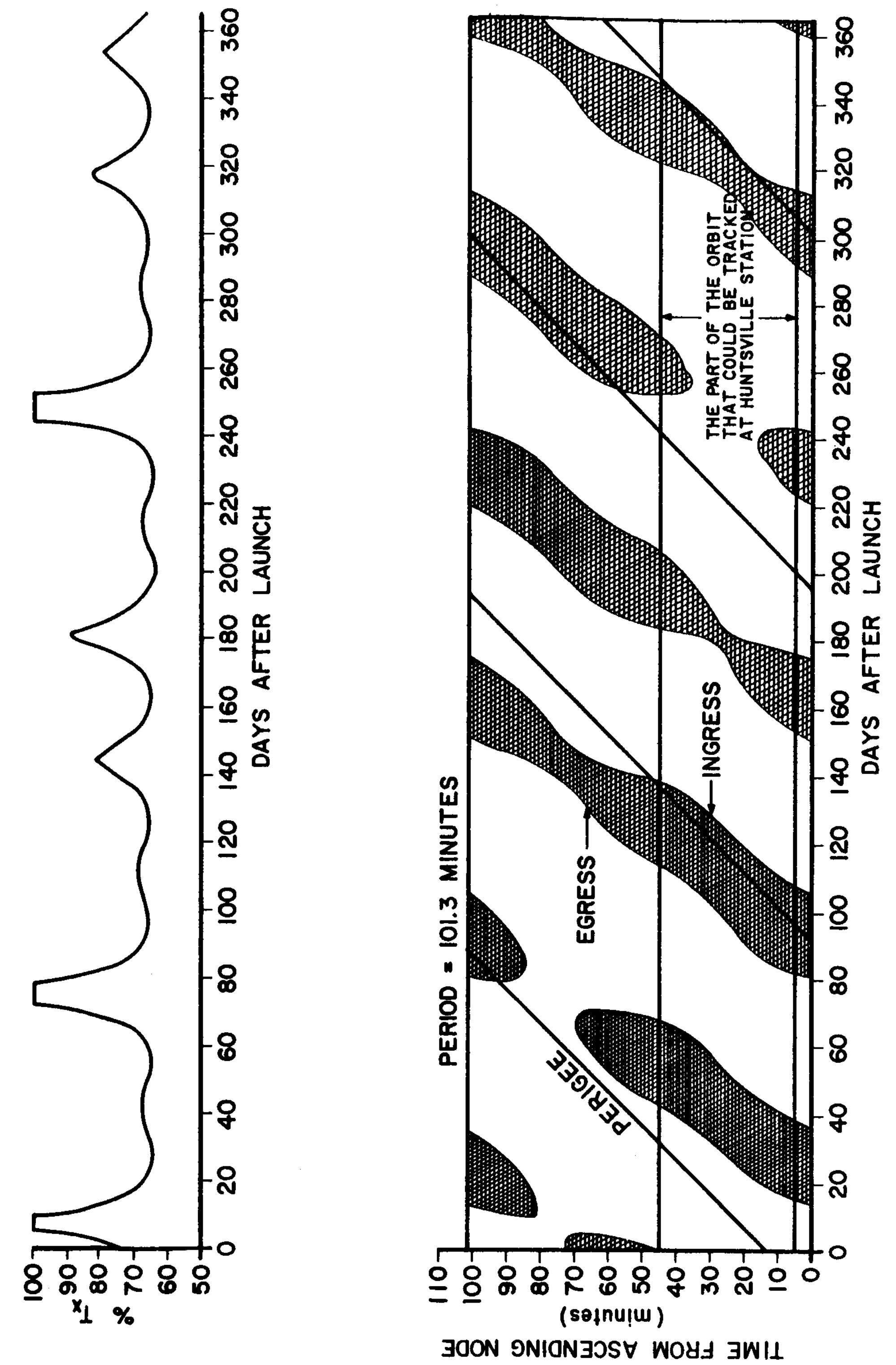


Fig. 14 Results for EXPLORER VII (IOTA I)

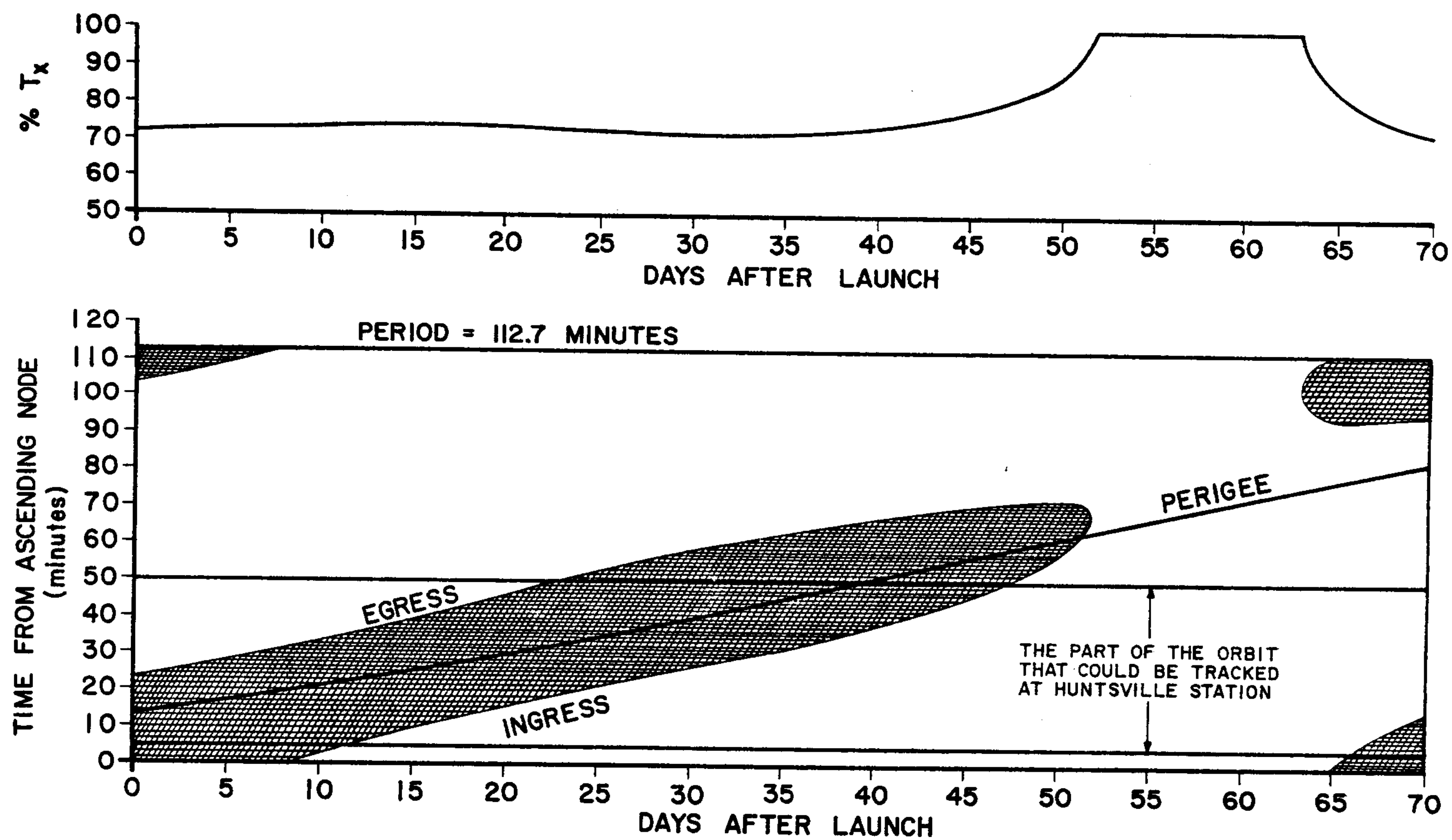


Fig. 15 Results for EXPLORER VIII
(1960 XI)

```
SHADOW CALCULATIONS
INITIAL CONDITIONS
TO     +55280000+01   DC     +22700000+03   ECC    +12100000+00   LON    +69600000+02   LAT    +37000000+02
I      +49500000+02   S      +10000000+01   RP     +67860000+04   RO     +63780000+04   EPS    +23450000+02
TMAX   +15000000+02   DT     +10000000+01

T      +00000000-37   TX     +71608004+02   TP     +12990850+02   TI     +10329433+03   TE     +22740100+02
AS     +20553441+03   DELTA  +63516059+01   OMEGA  +15804073+02   W      +51884414+02   LATP   +36999998+02
PS     +15364999+03   E1     +35195264+02   E2     +28221237+03   V1     +39413309+02   V2     +27533127+03
M1     +31199449+02   M2     +28898827+03   X1     +23681641+02   X2     +32236328+02   PHI    +63034949+02
LC.    +22012082+03   ALFHA  +21770757+03   GAMMA  -14087177+02   OMDOT  -34197925+01   WDOT   +28427665+01
ZP     +96891317+02   E      +45632482+02   P      +18748887+01   Z      +53758290+02   HO     +11655456+01
SIGMA  -72516771+02   OMCO   +32061327+03   OMEO   +15804071+02   ALFO   +21770757+03   WO     +51984414+02

T      +10000000+01   TX     +71777105+02   TP     +13746517+02   TI     +10462391+03   TE     +23879356+02
AS     +21033569+03   DELTA  +98557278+01   OMEGA  +12384281+02   W      +54727180+02   LATP   +38644816+02
PS     +15560651+03

T      +20000000+01   TX     +71910667+02   TP     +14509138+02   TI     +10561149+03   TE     +24715787+02
AS     +21372448+03   DELTA  +12487464+02   OMEGA  +89644889+01   W      +57569946+02   LATP   +40212882+02
PS     +15615453+03

T      +30000000+01   TX     +72060993+02   TP     +15279052+02   TI     +10664148+03   TE     +25577678+02
AS     +21723196+03   DELTA  +15005286+02   OMEGA  +55446968+01   W      +60412716+02   LATP   +41696065+02
PS     +15681925+03

T      +40000000+01   TX     +72242761+02   TP     +16056587+02   TI     +10772890+03   TE     +26460613+02
AS     +22087039+03   DELTA  +17394424+02   OMEGA  +21249046+01   W      +63255431+02   LATP   +43085707+02
PS     +15761491+03

T      +50000000+01   TX     +72436606+02   TP     +16842066+02   TI     +10886513+03   TE     +27376787+02
AS     +22465012+03   DELTA  +19639132+02   OMEGA  +35870510+03   W      +66098248+02   LATP   +44372706+02
PS     +15855187+03

T      +60000000+01   TX     +72642127+02   TP     +17635810+02   TI     +11005227+03   TE     +28334725+02
AS     +22857903+03   DELTA  +21722777+02   OMEGA  +35528531+03   W      +68941013+02   LATP   +45547643+02
PS     +15963602+03

T      +70000000+01   TX     +72853169+02   TP     +18438120+02   TI     +11128002+03   TE     +29325069+02
AS     +23266172+03   DELTA  +23628057+02   OMEGA  +35186552+03   W      +71783779+02   LATP   +46600967+02
PS     +16087794+03

T      +80000000+01   TX     +73060584+02   TP     +19249295+02   TI     +60526848-01   TE     +30365572+02
AS     +23685865+03   DELTA  +25337281+02   OMEGA  +34844572+03   W      +74626544+02   LATP   +47523233+02
PS     +16227210+03

T      +90000000+01   TX     +73257551+02   TP     +20069619+02   TI     +13619766+01   TE     +31445447+02
AS     +24128526+03   DELTA  +26832815+02   OMEGA  +34502593+03   W      +77469314+02   LATP   +48305403+02
PS     +16381594+03

T      +10000000+02   TX     +73439161+02   TP     +20899365+02   TI     +26934652+01   TE     +32572635+02
AS     +24581134+03   DELTA  +28097643+02   OMEGA  +34160614+03   W      +80312081+02   LATP   +48939187+02
PS     +16549926+03

T      +11000000+02   TX     +73590117+02   TP     +21738788+02   TI     +40390558+01   TE     +33748412+02
AS     +25046050+03   DELTA  +29116047+02   OMEGA  +33818635+03   W      +83154846+02   LATP   +49417402+02
PS     +16730565+03
```

NOMENCLATURE FOR COMPUTER RESULTS

Initial Conditions

TO	=	Universal Time during initial perigee passage
DO	=	Number of days after vernal equinox of initial perigee passage
ECC	=	Eccentricity of orbit
LON	=	Longitude of initial perigee (^{o} W)
LAT	=	Latitude of initial perigee
I	=	Inclination of orbit
S	=	Sign of north velocity component at initial perigee (+1 or -1)
RP	=	Radius of perigee (Km)
TMAX	=	Number of days to be run
DT	=	Number of days per increment

Output for each increment (all angles given in degree measurement)

T	=	Number of days from initial perigee
TX	=	Per cent time satellite spends in sunlight per orbit
TP	=	Time from ascending node to perigee in minutes
TI	=	Time from ascending node to ingress in minutes
TE	=	Time from ascending node to egress in minutes

AS	=	Angle between ascending node and sun in plane of orbit
DELTA	=	Angle between sun and plane of orbit
OMEGA	=	Right ascension of ascending node in plane of equator
W	=	Argument of perigee
LATP	=	Latitude of perigee
PS	=	Angle between perigee and sun in plane of orbit

<u>Addition output for first increment only</u>

E1	=	Eccentric anomaly of egress
E2	=	Eccentric anomaly of ingress
V1	=	True anomaly of egress
V2	=	True anomaly of ingress
M1	=	Mean anomaly of egress
M2	=	Mean anomaly of ingress
X1	=	Angle between egress and the twilight line in plane of orbit
PHI	=	Angle between perigee and twilight in plane of orbit
LC	=	Right ascension of sun in plane of ecliptic (longitude of sun)
ALPHA	=	Right ascension of sun in plane of equator
GAMMA	=	Angle between the sun and the plane of the equator

OMDOT	=	Regression rate of the ascending node in degrees per day
WDOT	=	Advance rate of argument of perigee in degrees per day
ZP	=	Angle between plane of orbit and the twilight line
B	=	Angle between orbit and equator in plane of twilight line
P	=	Period from ascending node to ascending node (hours)
Z	=	Space-fixed azimuth of perigee
HO	=	True sun time at perigee
SIGMA	=	Angle between twilight line and perigee in plane of equator
OMOO	=	Angle between perigee and ascending node in plane of equator
OMEO	=	Right ascension of ascending node in plane of equator
ALPO	=	Right ascension of sun in plane of equator
WO	=	Argument of perigee initially

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